Covariantising the Beltrami equation in W-gravity

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Abstract

Recently, certain higher dimensional complex manifolds were obtained in [1] by associating a higher dimensional uniformisation to the generalised Teichmüller spaces of Hitchin. The extra dimensions are provided by the "times" of the generalised KdV hierarchy. In this paper, we complete the proof that these manifolds provide the analog of superspace for W-gravity and that W-symmetry linearises on these spaces. This is done by explicitly constructing the relationship between the Beltrami differentials which naturally occur in the higher dimensional manifolds and the Beltrami differentials which occur in W-gravity. This also resolves an old puzzle regarding the relationship between KdV flows and W-diffeomorphisms.

Dedicated to the memory of Claude Itzykson

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1 Introduction

W-algebras have provided a unifying ground for diverse topics like integrable systems, conformal field theory, uniformisation and 2-dimensional gravity. It was originally discovered as a natural generalisation of the Virasoro algebra by Zamolodchikov[2] and implicitly in the work of Drinfeld and Sokolov who obtained (classical) W-algebras by equipping the "coefficients" of first order matrix differential operators with the second Gelfand-Dikii Poisson bracket[3]. As a result, one can now associate a W-algebra for every principal embedding of SL(2) in a semi-simple Lie group.

In the study of two dimensional gravity in the conformal gauge, it is well known that the generators of the residual reparametrisation invariance reduce to two copies of the Virasoro algebra. W-gravity can be defined as a generalisation of reparametrisation invariance such that in the "conformal gauge" one obtains two copies of the corresponding W-algebra. In W-gravity, the matrix differential operator is supplemented by another equation which is usually referred to as the *Beltrami equation*. The Beltrami equation is generically a complicated non-linear equation. These two equations can be rewritten as flatness conditions of a connection on a semi-simple group[4].

In a different context, Hitchin constructed certain generalised Teichmüller spaces associated to principal embeddings of SL(2) in a semi-simple Lie group[5]. These Teichmüller spaces arose as the moduli space of solutions of self-duality equations related to certain stable Higgs bundles. Recently, an explicit relationship has been obtained between these two constructions[6]. This was done by showing that the equivalence of the Teichmüller spaces constructed by Hitchin to the Teichmüller spaces for W-gravity.

Given this rich algebraic structure, it is natural to attempt a geometric picture for W-algebras. Various aspects of this issue have been tackled independently by many authors and is universally referred to as W-geometry[7, 4, 6]. This geometry is naturally related to W-gravity. For the case of Virasoro algebra, the geometric

structure has been studied extensively since the times of Poincaré and Klein and is related to the uniformisation of Riemann surfaces. Recently, a uniformisation in higher dimensions was shown to be related to the Teichmüller spaces constructed by Hitchin[1]. The extra dimensions are provided by a finite subset of the "times" of the generalised KdV hierarchy[1, 8]. In order to relate these higher dimensional manifolds to W-gravity, one has to obtain the relationship between KdV flows and W-diffeomorphisms. The fact that these two are not the same was first observed by Di Francesco *et al.*[9].

This relationship is addressed in this paper by studying the Beltrami equation on this higher dimensional setting. The Beltrami differentials which naturally occur here are related to KdV flows and not to W-diffeomorphisms. In this paper, we explicitly construct the relationship between the two sets of Beltrami differentials. The construction is based on the fact that the Beltrami differentials of W-gravity transform as tensors on the Riemann surface, unlike the higher dimensional Beltrami differentials.

The paper is organised as follows. In section 2, we discuss the results of Di Francesco *et al.* who covariantised the matrix differential operator of Drinfeld and Sokolov. In section 3, we briefly discuss higher dimensional uniformisation and how higher dimensional generalisations of Riemann surfaces are constructed. In section 4, we introduce the Beltrami equation in the higher dimensional setting and then using techniques described in section 2 to the Beltrami equation, we relate the two sets of Beltrami differentials.

2   Covariantising the DS equation

The first order matrix differential equation of Drinfeld and Sokolov can be converted into an ordinary differential equation of higher order, or more generally into a system of differential equations (the DS equation). In this section, we discuss how the DS operators can be rewritten in a covariant form. This has been done by Di Francesco

et al. and we will present their results[9]. See also related work[10]. For simplicity, we shall concentrate on the case of A_{n-1} in this paper. In this case, the DS equation is an n-th order linear differential operator L of the form

$$L \equiv \partial_z^n + u_2(z)\, \partial_z^{n-2} + \cdots + u_n(z) \quad , \tag{2.1}$$

The differential equation associated with this operator is

$$L\, f = 0 \quad . \tag{2.2}$$

The vanishing of the coefficient of $\partial_z^{n-1}\, f$ in the above differential equation implies that the Wronskian of the solutions is a constant (i.e., independent of the variable z). This condition is consistent across charts on a Riemann surface, provided the Wronskian is a scalar. This implies that f is a $\frac{(1-n)}{2}$-differential. Then the operator L provides a map from $K^{(1-n)/2}$ to $K^{(1+n)/2}$, where K is the canonical (holomorphic) line bundle on the Riemann surface.

The form invariance of the operator L under coordinate transformations enables us to derive the transformation properties of the the u_i. However, they do not transform as nice tensors. Di Francesco *et al.* have worked out an invertible change of variables $u_i \to w_i$ such that w_i for $i > 2$ transform as i-differentials and $u_2 = w_2$ transform like the Schwarzian[9]. This change of variables is non-linear in u_2. We shall summarise their results here. Rather than directly construct the change of variables it is simpler to rewrite the differential operator L in terms of covariant operators $\Delta_k(w_k, u_2)$ for $k = 2, \ldots, n$ such that

$$L = \sum_{k=2}^{n} \Delta_k(w_k, u_2) \quad . \tag{2.3}$$

Each of these operators Δ_k provide (covariant) maps from $K^{(1-n)/2}$ to $K^{(1+n)/2}$. When $u_2 = 0$, one can write

$$\Delta_k(w_k, u_2 = 0) = \sum_{i=0}^{n-k} \alpha_{k,i}\, w_k^{(i)}\, \partial_z^{n-k-i} \quad , \tag{2.4}$$

where $w_k^{(i)} \equiv \partial_z^i w_k$. The covariance of the operator under Möbius transformations

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which preserve the $u_2 = 0$ condition fixes $\alpha_{k,i}$ to be

$$\alpha_{k,i} = \frac{\begin{pmatrix} k+i-1 \\ i \end{pmatrix} \begin{pmatrix} n-k \\ i \end{pmatrix}}{\begin{pmatrix} 2k+i-1 \\ i \end{pmatrix}} \tag{2.5}$$

Starting from $u_2 = 0$, one can make $u_2 \neq 0$ by means of a (non-Möbius) change of coordinate $z \to z(t)$. Then $u_2(t) = c_n S[z,t]$ where $c_n = (n^3 - n)/12$ and $S[z,t] = \frac{z'''}{z'} - \frac{3}{2}(\frac{z''}{z'})^2$ (Here $'$ refers to differentiating w.r.t. t.) is the Schwarzian. The Δ_k transform as

$$\Delta_k(w_k(t), u_2) = J^{(1+n)/2} \sum_{i=0}^{n-k} \alpha_{k,i} \ [(J^{-1}\partial_t)^i J^{-k} w_k][J^{-1}\partial_t]^{n-k-i} J^{(n-1)/2} \quad , \tag{2.6}$$

where $J \equiv \frac{dz}{dt}$. It is a non-trivial fact that the right hand side of the above expressing depends on J only through the combination which occurs in the Schwarzian. The proof of this statement is given in ref. [9]. By using the relationship between the Schwarzian and u_2, we obtain the complete u_2 dependence of Δ_k.

The $u_i \to w_i$ transformation can be now explicitly obtained by comparing the two sides of eqn. (2.3). One obtains

$$w_k = \sum_{s=0}^{k-2} \beta_{k,s} u_{k-s}^{(s)} + \text{non} - \text{linear terms involving } u_2 \quad , \tag{2.7}$$

where

$$\beta_{k,s} = (-)^s \frac{\begin{pmatrix} k-1 \\ s \end{pmatrix} \begin{pmatrix} n-k-s \\ s \end{pmatrix}}{\begin{pmatrix} 2k-2 \\ s \end{pmatrix}} \quad .$$

We refer the reader to ref. [9] for more details.

3 Higher Dimensional Uniformisation

In this section, we will briefly describe the construction of higher dimensional generalisation of Riemann surfaces associated with $\mathrm{PSL}(n, \mathbf{R})$. The reader is referred to [1] for more details.

It is a well known result that Riemann surfaces of genus $g > 1$ can be obtained by the quotient of the upper half plane (with Poincaré metric of constant negative curvature) by a Fuchsian subgroup of $\mathrm{PSL}(2, \mathbf{R})$. This is usually referred to as the Uniformisation Theorem. The space of all Fuchsian groups furnishes the Teichmüller space of Riemann surfaces. Hitchin generalised these Teichmüller spaces by replacing $\mathrm{PSL}(2, \mathbf{R})$ with any semi-simple group[5]. In recent work[1], certain generalisations of Riemann surfaces were constructed whose Teichmüller spaces are precisely those of Hitchin.

The method employed in [1] is a generalisation of the differential equation approach to uniformisation as studied by Poincaré and Klein in the 1880's. Poincaré considered the second order Fuchsian linear differential equation on a Riemann surface Σ of genus $g > 1$

$$[\partial_z^2 + u_2(z)] \, f = 0 \quad , \tag{3.1}$$

where $u_2(z)$ have no singularities[2]. This corresponds to $n = 2$ in eqn. (2.2).

Locally (on a chart), the differential equation has two linearly independent solutions f^1 and f^2, which can be considered as homogeneous coordinates on CP^1. On analytically continuing the solutions along any of the cycles of the Riemann surface, the solutions go into linear combinations of each other. Thus one can associate a monodromy matrix to each cycle which encodes this mixing. The analyticity of the solutions implies that this matrix depends only on the homotopy class of cycle. The set of monodromy matrices corresponding to each element of $\pi_1(\Sigma)$ form the monodromy group Γ of the differential equation (which is isomorphic to $\pi_1(\Sigma)$). Generically, one obtains that the monodromy group is a subgroup of $\mathrm{PSL}(2, \mathbf{C})$ (provided we normalise the basis such that its Wronskian is 1). However, we shall restrict ourselves to those differential equations which have a monodromy group Γ which is a Fuchsian group and hence a subgroup of $\mathrm{PSL}(2, \mathbf{R})$.

The maps from Σ to CP^1 are thus not single valued and hence when one changes

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[2]Regular singular points correspond to punctures and will not be considered here.

charts, there is a PSL(2, **R**) matrix which changes the basis. Such maps are called *polymorphic* and this multivaluedness encodes the monodromy data of the differential equation. Using a standard trick, one can globalise the patch data on the Riemann surface by lifting the differential equation to the universal cover of the Riemann surface. Now the differential equation gives a map from the universal cover to CP^1. Unlike before, now one obtains a nice one-to-one map with the monodromy group encoding the fundamental group of the underlying Riemann surface. When the monodromy group is Fuchsian, the image of the universal cover in CP^1 is the upper half plane. This follows from the fact that under the action of PSL(2, **R**), CP^1 splits into three parts – the upper half plane, the circle and the lower half plane. We can choose the image to be the upper half plane with no loss of generality. The monodromy group tesselates the upper half plane with $4g$-gons (whose sides are geodesics in the Poincaré metric). The $4g$-gon represents a Riemann surface of genus g just as the torus can be represented by a tesselation of the complex plane by a lattice. Thus one recovers the standard case of uniformisation of Riemann surfaces.

We shall now generalise this to the case of n-th order Fuchsian differential equations (again with no regular singular points) on Σ with the monodromy group given by an element of the Teichmüller space of Hitchin corresponding to the group PSL(n, **R**). Now one gets polymorphic maps from the Riemann surface to CP^{n-1}. Again, as before, we lift the differential to the universal cover of Σ. Let the image of the universal cover be Ω. Ω has complex dimension one like the Riemann surface and the monodromy group tesselates the image as before. However, this is not sufficient to create the higher dimensional manifold promised earlier. There exist a set of deformations of the differential equation which preserve the monodromy group (and hence the point in the Teichmüller space). These are the so called isomonodromic deformations which are parametrised by the times of the generalised KdV hierarchy[3].

$$\frac{\partial L}{\partial t_i} = [(L^{i/n})_+, L] \quad , \tag{3.2}$$

where $(L^{i/n})_+$ represents the differential operator part of the pseudo-differential op-

erator $L^{i/n}$ and $t_1 \equiv z$. Further, the u_i have been extended to be functions of all the times such that equation (3.2) is satisfied. Since only the first $(n-1)$ times furnish coordinates in $\mathrm{CP^{n-1}}$, we shall restrict to only those times.

Now, for each of these times, one obtains the image of the universal cover in $\mathrm{CP^{n-1}}$, which we shall call $\Omega(\{t_i\})$ for all t_i, $i = 2, \ldots, n-1$. Form the union $\tilde{\Omega} \equiv \cup \Omega(\{t_i\})$. The monodromy group Γ tesselates each of the time slices and hence all of $\tilde{\Omega}$. The higher dimensional manifold, which we shall call the *W-manifold* is obtained as $\tilde{\Omega}/\Gamma$. By construction, this has complex dimension $(n-1)$.

4   Covariantising the Beltrami Equation

In this section, we first introduce the Beltrami equation on the W-manifold. The Beltrami differentials which occur in this equation are naturally related to the generalised KdV flows. By projecting this equation onto the Riemann surface, we show how one can explicitly construct a relationship to the Beltrami differentials which occur in W-gravity. This solves the long standing puzzle regarding the relationship between W-diffeomorphisms and generalised KdV flows raised by Di Francesco *et al.*.

In the previous section, we had restricted ourselves to the holomorphic part and implicitly assumed that the f were holomorphic. The change of complex structure on the W-manifold is given by the Beltrami equation[1]

$$[\bar{\partial}_{\bar{z}} + \mu_{\bar{z}}{}^i \partial_i - \frac{1}{n}(\partial_i \mu_{\bar{z}}{}^i)] \ f = 0 \quad , \tag{4.1}$$

where $i = 1, \ldots, (n-1)$ and $\mu_i \equiv \mu_{\bar{z}}{}^i$ are the generalised Beltrami differentials.[3] On projecting this Beltrami equation to the Riemann surface, we obtain

$$[\bar{\partial}_{\bar{z}} + \mu_i (L^{i/n})_+ - \frac{1}{n}(\partial_i \mu_i)] \ f = 0 \quad , \tag{4.2}$$

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[3]In general, one would have expected complex deformations with respect to the other complex coordinates. In [1], it was however argued that the W-manifolds are rigid to such complex deformations and hence we will not consider them.

where we have used the relation $\partial_i f = (L^{i/n})_+ f$. However, the μ_i have complicated transformations under coordinate transformations on the Riemann surface. We shall derive new Beltrami differentials ρ_i from the μ_i such that they transform as $(-i, 1)$ differentials. The ρ_i are precisely the Beltrami differentials associated with W-gravity. As we shall explain below, this is the dual of the $u_i \to w_i$ transformation. We shall rewrite the projected Beltrami equation as follows

$$[\sum_{i=0}^{n-1} \mathcal{B}_i(\rho_i, w_2) + \text{terms involving } w_i \text{ for } i > 2] \, f = 0 \quad , \tag{4.3}$$

where $\mathcal{B}_0 \equiv \bar{\partial}_{\bar{z}}$. $\mathcal{B}_i$ are (covariant) differential operators which furnish maps from $K^{(1-n)/2}$ to $K^{(1-n)/2}\bar{K}$ and are constructed from ρ_i and w_2. Equating the coefficients of ∂_z^i for $i = 1, \ldots, (n-1)$ in eqns. (4.3) and (4.2), we obtain the $\mu_i \to \rho_i$ transformation. This is an invertible transformation. Further, we have assumed that this transformation is independent of w_i for $i > 2$. Towards the end of this section we shall provide evidence that this assumption is valid.

When $w_i = 0$, one has $(L^{i/n})_+ = \partial_z^i$ and $\mathcal{B}_i$ can be written as

$$\mathcal{B}_i(\rho_i, w_2 = 0) \equiv \sum_{s=0}^{i} \gamma_{i,s} \rho_i^{(s)} \partial_z^{i-s} \quad , \tag{4.4}$$

with $\gamma_{i,0} = 1$ fixing the normalisation of the ρ_i. Comparing equations (4.2) and (4.4), we obtain

$$\mu_j = \sum_{i \geq j} \gamma_{i,i-j} \rho_i^{(i-j)} + \text{ terms involving } w_2 \quad . \tag{4.5}$$

The unknown coefficients $\gamma_{i,j}$ are determined by requiring that $\mathcal{B}_i f$ transforms covariantly under Mobiüs transformations which preserve the $w_2 = 0$ condition. We obtain

$$\gamma_{i,s} = (-)^s \frac{\binom{n+s-i-1}{s} \binom{i}{s}}{\binom{2i}{s}} \tag{4.6}$$

Restoring the dependence on w_2 is now easy. Under arbitrary coordinate transformations, the $w_2 = 0$ condition is not preserved. Like in the case of Δ_k, all one has to do

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is to make an arbitrary coordinate transformation of the $\mathcal{B}_i(\rho_i, w_2 = 0)$ and identify the terms corresponding to the Schwarzian of the transformation with w_2. Further, the arguments in ref. [9] showing that Δ_k depend on J via the combination occuring in the Schwarzian is also valid for the $\mathcal{B}_i$. The $\mathcal{B}_i$ are given by

$$\mathcal{B}_i(w_k(t), w_2) = J^{(1-n)/2} \sum_{s=0}^{i} \gamma_{i,s} \, [(J^{-1}\partial_t)^s J^i \rho_i][J^{-1}\partial_t]^{i-s} J^{(n-1)/2} \quad , \qquad (4.7)$$

Explicitly, the first few $\mathcal{B}_i$ (with non-zero w_2) are

$$\begin{aligned}
\mathcal{B}_1 &= \rho_1 \partial_z - \frac{n-1}{2}\rho_1' \quad , \\
\mathcal{B}_2 &= \rho_2 \partial_z^2 - \frac{n-2}{2}\rho_2'\partial_z + \frac{(n-1)(n-2)}{12}\rho_2'' + \frac{2}{n}\rho_2 w_2 \quad , \\
\mathcal{B}_3 &= \rho_3 \partial_z^3 - \frac{n-3}{2}\rho_3'\partial_z^2 + (\frac{(n-2)(n-3)}{10}\rho_3'' + \frac{6(3n^2-7)}{5(n^3-n)}w_2\rho_3)\partial_z \\
&\quad - \frac{(n-1)(n-2)(n-3)}{120}\rho_3^{(3)} - \frac{(4n+7)(n-3)}{5n(n+1)}w_2\rho_3' - \frac{3(n+2)(n-7)}{10n(n+1)}w_2'\rho_3
\end{aligned}$$

$$(4.8)$$

Finally, by again comparing (4.3) with (4.2) with the w_2 also restored in $(L^{i/n})_+$, we obtain the complete $\mu_i \rightarrow \rho_i$ transformation[4]. The first few terms are

$$\begin{aligned}
\mu_1 &= \rho_1 - \frac{n-2}{2}\rho_2' + (\frac{(n-2)(n-3)}{10}\rho_3'' + \frac{3(n^2-9)}{5(n^3-n)}w_2\rho_3) + \cdots \\
\mu_2 &= \rho_2 - \frac{n-3}{2}\rho_3' + \cdots \\
\mu_3 &= \rho_3 + \cdots
\end{aligned}$$

$$(4.9)$$

These agree with the known expressions for the change of variables[1, 8]. Note that in the above expressions, for a given n, only the first $(n-1)$ ρ_i are non-vanishing. Hence to obtain the required transformation, one has to set the others to zero.

The argument which suggests that only w_2 can occur in the $\mu_i \rightarrow \rho_i$ transformation is as follows. The $u_i \rightarrow w_i$ transformation is linear for $w_2 = 0$. The Beltrami differentials are the conjugate variables to the projective connections w_i as given by

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[4]We have used $(L^{3/n})_+ = \partial_z^3 + \frac{3}{n}w_2\partial_z + \cdots$ in order to obtain the $w_2\rho_3$ term.

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the following natural symplectic form[5] on the space (ρ_i, w_i)[11]

$$\Omega = \frac{1}{2\pi\hbar} \sum_{i=2}^{n} \int (-)^i \delta\rho_i \wedge \delta w_i \quad .$$

For $w_2 = 0$, the change of variables is nothing but a change of basis. The $\mu_i \to \rho_i$ transformation can then be obtained from the $u_i \to w_i$ transformation using the invariance of the symplectic form Ω. One can check that $\sum(-)^i \mu_{i-1} u_i = \sum(-)^i \rho_{i-1} w_i$ up to total derivatives using the expressions derived earlier. Further, since the non-linear terms in $u_i \to w_i$ transformation only involve w_2 it seems likely that this would also be true for the $\mu_i \to \rho_i$ transformation. Thus, the w_i for $i > 2$ cannot occur in the $\mu_i \to \rho_i$ transformation.

The relationship between KdV flow and W-diffeormorphisms is provided in eqn. (4.9) by identifying $\rho_i = \bar{\partial}_{\bar{z}} \epsilon^i$ and $\mu_i = \bar{\partial}_{\bar{z}} \tilde{\epsilon}^i$, where ϵ^i parametrise infinitesimal W-diffeomorphisms and $\tilde{\epsilon}^i$ parametrise infinitesimal KdV flows. One sees that $\epsilon^i = \tilde{\epsilon}^i$ for constant flows provided $w_2 = 0$. This resolves the issue raised in [9]. Further, as conjectured in [1] for the case of W_4, one can see that w_2 enters the change of variables.

In conclusion, we have presented the relationship between the higher dimensional manifolds constructed in [1] and W-gravity. As a by product, we have obtained an algorithm to covariantise the Beltrami equation. Even though we have restricted to the case of A_n, the generalisation to other groups is straightforward.

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[5]The $(-)^i$ in the definition is essential in order to agree with the standard definitions of ρ_i and w_i. However, there is a Z_2 invariance of the W-algebra given by $w_i \to (-)^i w_i$. So one can absorb the $(-)^i$ into the w_i which would correspond to a non-standard choice.

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